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Basic Applied Reservoir Simulation

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Dedication

**This book is dedicated to our mentors and friends,
C. Drew Stahl and S.M. Farouq Ali.**

Introduction

High-speed computers have now been part of our lives, in so many different ways, for almost half a century. The electronic explosion that we have been witnessing over the past two decades has transformed reservoir simulation from a somewhat esoteric approach to a practical toolbox of immense importance. With the use of the tools from this toolbox, today's engineering community has an opportunity to better understand not only the intricacies of fluid-flow dynamics in increasingly complex reservoirs, but also the characteristics of fluid-flow dynamics in wellbores, flow patterns developing within the immediate vicinity of perforations, the interaction of vertical, slanted, horizontal, and multilateral wells and the reservoir, and the complexities of reservoir characterization. These areas represent only a small portion of a list that includes some challenging issues playing critical roles in the optimum development of hydrocarbon reservoirs, and in the optimized implementation of capital-intensive projects that can be investigated with numerical simulation.

The conduct of a reservoir-simulation study should not simply imply making a few computer runs and writing a report based on the computer-generated results. In our view, the conduct of a simulation study covers much more. First, the simulation engineer must set the objectives for the study. Envisaging a judicious set of objectives will assist the simulation engineer in selecting an adequate approach that is in parity with the scope of the study as well as with the characteristics of the reservoir and its fluids. The third step of the process involves the preparation of the input data. The time invested in looking for good-quality data represents time well spent; an internally consistent set of data will save a great deal of time later. The fourth step of the simulation study involves careful planning of computer runs. It is again the simulation engineer's responsibility to ensure that each run conducted does not represent a "shot in the dark." The final step of a simulation study involves the analysis of results and report preparation. An experienced simulation engineer will not subscribe immediately to the results presented within the output files. To avoid becoming a hostage to computer-generated results, it is necessary to ask questions and ponder the implications of those results. Therefore, it is very important to remember that every simulation study carries the signature of the simulation engineer, but not the computing device and the computer code used in the study.

*I guess the take-home lesson from it all is that the most important thing for the success of an exhaustive simulation study is not the hardware or software... It's the individuals from different disciplines that you work with. If the people are compatible and can get along, then it will be a successful simulation study.**

Preface

This text is written for senior undergraduate students and first-year graduate students studying petroleum engineering. The text evolved from the courses that we presented in university settings. The examples and exercises presented are from the examinations and homework sets that we prepared over the course of several years. The contents of this book can be taught in three successive courses. In our own teaching experience, we were able to cover various single-phase reservoir simulation topics and applications, included mainly in Chaps. 1 through 8, during a one-semester undergraduate senior-level course. We have followed a similar coverage, in greater depth, in the first graduate-level course. The second graduate-level course dwells mainly on multiphase-flow-simulation problems, as covered in Chap. 9. A third course, which deals mainly with the practical application of reservoir simulation (as covered in Chap. 10), can be given as either an undergraduate senior-level course or as a graduate course to practicing engineers.

Chap. 1 provides an overview of numerical reservoir simulation. In Chap. 2, we present a brief review of some fundamental reservoir engineering concepts as well as reservoir rock and fluid properties that comprise the building blocks of a reservoir simulator. Chap. 3 is written to serve as a refresher in mathematics and presents an introductory treatment of finite-difference calculus as it forms the backbone of reservoir simulation. In this chapter, we also hope to establish a bridge between mathematical reasoning and jargon and reservoir-engineering concepts. In Chap. 4, rectangular and cylindrical coordinate systems are introduced and various forms of the single-phase flow equations are developed. In Chap. 5, protocols used in obtaining the finite-difference analogs of linear-flow equations are discussed. Chap. 6 introduces various well models and their coupling to the reservoir-flow equations. Some direct and iterative algorithmic protocols, used in solving linear-difference equations presented in their increased rigor, are discussed in Chap. 7. In Chap. 8, transmissibility groups are defined and coefficient matrices for incompressible, slightly compressible, and compressible flow problems are formed. After solving the system of equations, incremental and cumulative material-balance checks are introduced as internal checks that monitor the accuracy level of the solutions generated. Chap. 9, in its entirety, is devoted to multiphase flow and its simulation. In this chapter, procedures and algorithms introduced in the first eight chapters are generalized so that they become applicable to multiphase-flow problems. In Chap. 10, our intent is to bring the practical aspects of reservoir simulation to the forefront. Topics such as data analysis, model construction, history matching, and forecasting are discussed. Finally, Chap. 11 ties the reservoir simulation equations back to classical reservoir engineering approaches and shows that the latter are simply the subsets of the former. The book concludes with three appendices. Appendix A provides a thorough treatment of interpolation techniques that are often used in reservoir simulation for data handling. Appendix B shows the similarities between the single-phase and multiphase flow problems at the level of the coefficient matrices generated by the finite-difference representation. Appendix C presents a brief overview of the architectures of scalar, vector, and parallel processors.

Our discussion of the topical material in each chapter typically concludes with a section identified as "Chapter Project." Starting with Chap. 1, these sections provide a large field-scale example. In this way, we are able to construct a field example throughout the book. As the reader progresses through the chapters of the book, the chapter projects will provide an opportunity to apply some of the salient topics discussed in each chapter in a more realistic setting. Furthermore, the discussions and results provided through this "marching" example can be used as benchmark solutions if the reader is engaged in reservoir simulator development.

* Adapted from astronaut Shannon Lucid's statement upon her return to earth after 6 months aboard Russia's *Mir* space station. Her original statement reads, "I guess the take-home lesson from it all is that the most important thing for the success of a long-duration space flight is not the hardware... It's the people you fly with. If the people are compatible and can get along, then it will be a great flight."

Throughout the book, numerous examples that have been specifically designed to be solved by hand calculations are dispersed. With these examples, our goal is to create an opportunity for the reader to better and more effectively understand some of the fine details of reservoir simulation. The additional exercises are designed to redrill basics and improve the reader's understanding as well as to test their innovation for more difficult problems.

When we started this book, we neither wanted to produce a handbook on reservoir simulation, nor did we envisage how to write a user's manual that comes together with a simulation package. Our goal was to create a textbook that would help in breaking the ice between neophyte simulation students (or engineers) and the mathematical nature of reservoir simulation. In our simulation courses, we always draw an analogy between developing a reservoir model and raising a child. The time and effort expended during the development phase of a simulator would determine our expectations for the simulator. We hope that the readership of this book will acquire basic understanding of the mechanics involved in developing and applying reservoir simulators. In our classrooms, we have always thought that students who go through the rigor of developing a reservoir simulator, even a simple one, would develop a much better appreciation for the strengths and weaknesses of the tool. Thus, even if they are confronted with the request of only implementation of a predeveloped simulation package (not its modification), they will be able to demonstrate that they are much more informed and confident users.

In this book the reader will find that our approach is to proceed through the various stages of model development so that solutions to the problems in increasingly more complicated domains can be sought. In this way, we create opportunities for the treatment and discussion of more sophisticated procedures and algorithms. Over the years, for pedagogical reasons, we have found the modeling "from the bottom up" approach to be effective in a classroom environment. We hope that the readers of this book will agree with our philosophy.

We would like to close this preface with a quotation from *Calculus Made Easy* by Silvanus P. Thompson:**

Considering how many fools can calculate, it is surprising that it should be taught either a difficult or a tedious task for any other fool to learn how to master the same tricks. Some calculus tricks are quite easy. Some are enormously difficult. The fools who write the textbooks of advanced mathematics—and they are most clever fools—seldom take the trouble to show you how easy the calculations are. On the contrary, they seem to desire to impress you with their tremendous cleverness by going about it in the most difficult way. Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

**Turgay Ertekin
Jamal H. Abou-Kassem
Gregory R. King**

**Thompson, S.P. and Martin Gardner, M.: *Calculus Made Easy*, St. Martin's Press, New York City (1998) 38. The original edition of *Calculus Made Easy* was written by Silvanus P. Thompson and published in 1910, with subsequent editions in 1914 and 1946.

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We have enjoyed the pleasure of teaching the material covered in this text to students at The Pennsylvania State U. and the United Arab Emirates U. for more than two decades. We gratefully acknowledge their patience, suggestions, and comments, all of which have been instrumental in bringing this book to reality.

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Finally, but most deeply, we thank all of our "teachers," from whom we have learned all that we know, and our family members, who walked with us through the tortuous paths of writing a book. Without their constant encouragement, support, and most importantly their tolerance, the journey never would have ended.

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Chapter 1

Introduction

1.1 Introduction

Reservoir simulation combines physics, mathematics, reservoir engineering, and computer programming to develop a tool for predicting hydrocarbon-reservoir performance under various operating conditions. This book is limited to the basics of this subject and is aimed at developing understanding of and insight into the mechanics of this powerful tool. Chap. 1 presents a review of the prediction techniques available to petroleum engineers, with an emphasis on practical limitations. To develop appreciation for the role of reservoir simulation in optimizing the development and production of hydrocarbon resources, the chapter also presents an overview of reservoir simulation and its applications in hydrocarbon recovery.

1.2 The Need for Reservoir Simulation

The need for reservoir simulation stems from the requirement for petroleum engineers to obtain accurate performance predictions for a hydrocarbon reservoir under different operating conditions. This need arises from the fact that in a hydrocarbon-recovery project (which may involve a capital investment of hundreds of millions of dollars), the risk associated with the selected development plan must be assessed and minimized. Factors contributing to this risk include the complexity of the reservoir because of heterogeneous and anisotropic rock properties; regional variations of fluid properties and relative permeability characteristics; the complexity of the hydrocarbon-recovery mechanisms; and the applicability of other predictive methods with limitations that may make them inappropriate. The first three factors are beyond the engineer's control; they are taken into consideration in reservoir simulation through the generality of input data built into reservoir-simulation models and the availability of simulators for various enhanced-oil-recovery techniques. The fourth factor can be controlled through proper use of sound engineering practices and judicious use of reservoir simulation.

1.3 Traditional Modeling Approaches

Traditional methods of forecasting reservoir performance generally can be divided into three categories: analogical methods, experimental methods, and mathematical methods. Analogical methods use properties of mature reservoirs that are either geographically or petrophysically similar to the target reservoir to attempt to predict reservoir performance of a target zone or reservoir. Experimental methods measure physical properties (such as rates, pressures, or saturations) in laboratory models and scale these results to the entire hydrocarbon accumulation. Finally, mathematical methods use

equations to predict reservoir performance. The remaining sections of this chapter provide more detailed discussions of these methods.

1.3.1 Analogical Methods. Before drilling, when limited or no data are available, the only method reservoir engineers can use to perform economic analysis is that of analogy. In this method, reservoirs in the same geologic basin or province or reservoirs with similar petrophysical properties are used to predict the performance of the target reservoir. This method can be used to estimate recovery factors, initial production rates, decline rates, well spacing, and recovery mechanisms. The analogical method can yield reliable results when two similar reservoirs are compared and similar development strategies are used. The method suffers, however, if different development strategies are considered. In addition, "what-if" sensitivities cannot be investigated.

One form of analogy, the staged field trial, provides the most reliable predictions for secondary- and tertiary-recovery operations. In this method, representative well patterns in a field that is a candidate for secondary or tertiary recovery are converted to the new process and the production performance is monitored. The results of the field trial, which may take 1 or 2 years to obtain, are applied to the remaining well patterns, and field performance can be predicted. Managements are generally confident with decisions made on the basis of results of a staged field test.

1.3.2 Experimental Methods. Experimental methods, both analog and physical, play a key role in understanding petroleum reservoirs. While analog models are seldom used today, physical models in the form of corefloods, sandpacks, and slim tubes are run often.

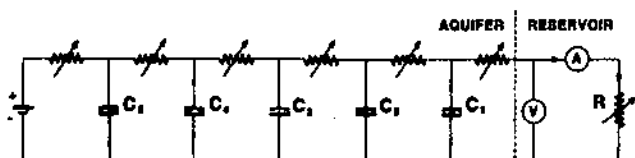
Analog Models. Analog models are rarely used in modern reservoir studies, but two points about them are worthy of discussion. First, from a historical point of view, analog models were important in early studies, particularly in incorporating sweep efficiencies into waterflood calculations. Second, the difference between resistance-capacitance (RC) networks and potentiometric models illustrates the difference between discrete and continuous models.

Analog models use similarities between the phenomenon of fluid flow through porous media and other physical phenomena (such as those Table 1.1 shows) to simulate reservoir performance. Analog models based on the governing equations listed in the table are built to represent the reservoir, and the appropriate quantities (those representing pressure and flow rate) are measured. These quantities can be translated through the governing equations into their porous-medium analogs. Three analog methods—RC networks, potentiometric models, and the Hele-Shaw¹ models—are discussed next.

TABLE 1.1—PHYSICAL PHENOMENA ANALOGOUS TO FLUID FLOW THROUGH POROUS MEDIA

Phenomenon	Fluid Flow Through Porous Media	Fluid Flow Through Parallel Plates	Electricity Flow Through Circuitry	Heat Flow by Conduction
Governing equation	Darcy's law* $q = \frac{\beta_c k A}{\mu} \frac{\Delta p}{\Delta L}$	Hagen-Poiseuille law $q = \frac{w^2 A}{12 \alpha_c \mu} \frac{\Delta p}{\Delta L}$	Ohm's law, $I = (1/R) \Delta E$	Fourier's law, $Q = \frac{KA \Delta T}{\Delta L}$
Properties	Volumetric rate, q Transmissibility, $\frac{\beta_c k A}{\mu \Delta L}$ Fluid mobility, $\frac{\beta_c k}{\mu}$ Pressure, p	Volumetric rate, q Hydraulic conductance, $\frac{w^2 A}{12 \alpha_c \mu \Delta L}$ Hydraulic conductivity, $\frac{w^2}{12 \alpha_c \mu}$ Pressure, p	Current, I Electrical conductance, $1/R$ Electrical conductivity,** $1/r$ Voltage, E	Heat flow rate, Q Thermal conductance, $\frac{KA}{\Delta L}$ Thermal conductivity, K Temperature, T

*Horizontal flow.

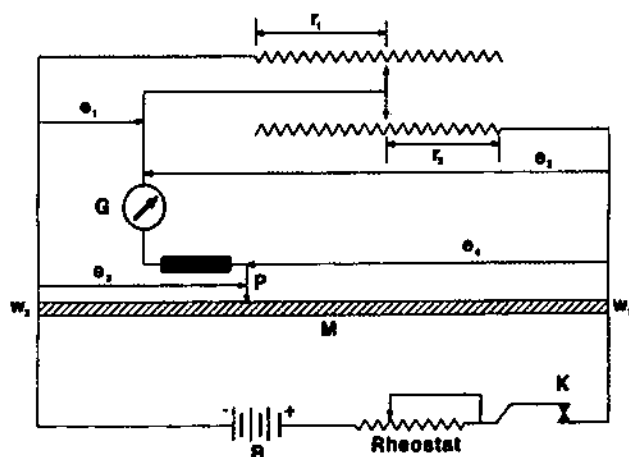
** $R = \frac{\rho \Delta L}{A}$.Fig. 1.1—Electric-circuit analog for a simple hydrocarbon reservoir/aquifer system.³ (C_1 through C_5 = capacitors; R = resistor; A = ammeter; and V = voltmeter.)

RC networks use the analogy between fluid flow through porous media and electrical flow to model reservoir performance. Bruce² introduced this method to the petroleum industry to simulate the unsteady-state performance of undersaturated oil reservoirs under waterdrive. Fig. 1.1 shows the RC network for this problem. In these models, capacitance is used to model fluid storage at a point in space, while resistance is used to model the transmissibilities between points. Capacitor discharge represents the unsteady-state behavior of the reservoir in accordance with the properties listed in Table 1.1. As a final note on RC networks, although these circuits simulate unsteady-state behavior (and, therefore, may represent reservoirs undergoing primary depletion), they are discrete models. That is, the capacitors represent the storage at discrete points in the reservoir.

A continuous form of the electrical analog is the potentiometer. A potentiometer is a scaled model of a reservoir or well pattern constructed with a continuous electrical conducting material. Voltages are applied at wellsites and voltage measurements can be made at any point within the model. This is in contrast to the RC circuit, where measurements can be made at only discrete points in the reservoir. A second difference between RC networks and potentiometers is that potentiometers can simulate only steady-state flow. Most early studies on sweep efficiencies of waterflood patterns were conducted on models like that depicted in Fig. 1.2.

In general, electrical analog models must be custom built for individual reservoirs, making them very difficult to adapt to other reservoirs. The discrete RC-network models also suffer from inadvertent malfunction of electrical components (capacitors, meters, resistors) and the huge space they usually occupy (several rooms). Aside from these deficiencies, these analogs are limited to modeling single-phase flow in porous media or, at best, two-phase flow with a unit mobility ratio.

The Hele-Shaw¹ model is an analog model that allows for nonunit mobility ratios. Hele-Shaw models use the analogy between fluid flow through porous media and fluid flow between parallel plates to simulate the behavior of regular pattern elements in secondary- and tertiary-recovery operations. These models are constructed with two transparent plates spaced at a uniform distance from each other. The gap between the plates is filled with the fluid to be displaced, while the displacing fluid is introduced at the injection wellsites. The sweep efficiencies of the reservoir patterns are then determined visually.

Fig. 1.2—Electrical circuit for determining the potential distributions in 2D flow systems.⁴ (B = battery; r_1 and r_2 = resistors; G = galvanometer; e_1 through e_4 = potential drops; M = potentiometer; w_1 and w_2 = electrodes representing the injectors and producers; P = exploring electrode; and K = key.)

Physical Models. As opposed to analog models, physical models are used to make direct measurements of flow properties in porous media. Two types of physical models are in use in the petroleum industry. The first does not account for the flow geometry occurring in the reservoir. Coreflood experiments fall into this category. These experiments, generally run on linear cores, are probably the most common physical models used in the oil industry today. They are run on virtually every oil and gas field to determine reservoir properties, such as porosity and permeability, and to establish mechanisms of oil recovery. One detrimental feature of these models is that the experiments are conducted at a scale that is not representative of actual reservoir scale. Consequently, the results of these experiments must be scaled up to more representative scales. Other physical models that fall into this category include slim tubes and sandpicks.

The second type of physical model uses geometrical-, mechanical-, and thermal-similarity concepts. That is, the areal geometry, thickness, porosity, and permeability of the model and the fluid properties are scaled so that the shape and dimensions of the model (as well as the ratios of active forces in the model) are the same as those in the reservoir. The performance of this type of scaled model reflects that of the reservoir. One example is Sobocinski and Cornelius's⁵ single-well coning model (Fig. 1.3). This type of model can determine critical coning rates, water-breakthrough times, and post-breakthrough water cuts. Note, however, that, in reservoir-engineering problems, it is generally impossible to scale all physical characteristics of the reservoir, so the use of truly scaled models is very limited. Adequately